Specimen Paper WMA12/01

Question Number	Scheme	Marks
1. (a)	$\left(1-\frac{1}{8}x\right)^{10}=1+10\left(-\frac{1}{8}x\right)^{1}+\frac{10\times 9}{2}\left(-\frac{1}{8}x\right)^{2}+\frac{10\times 9\times 8}{3!}\left(-\frac{1}{8}x\right)^{3}+\ldots$	M1
	$=1-\frac{5}{4}x+\frac{45}{64}x^{2}-\frac{15}{64}x^{3}$	A1, A1
		(3)
(b)	$\mathrm{g}(x)=(3+10x)\left(1-\frac{5}{4}x+\frac{45}{64}x^{2}-\frac{15}{64}x^{3}+\right)$	
	Coefficient of $x^3 = 10\times\frac{45}{64}+3\times-\frac{15}{64}=\frac{405}{64}$	M1 A1
		(2) **(5 marks)**

(a)

M1 For an attempt at the binomial expansion. Score for a correct attempt at term 2, 3 or 4.

Accept sight of ${}^{10}\mathrm{C}_1\left(\pm\frac{1}{8}x\right)^{1}$ or ${}^{10}\mathrm{C}_2\left(\pm\frac{1}{8}x\right)^{2}$ or ${}^{10}\mathrm{C}_3\left(\pm\frac{1}{8}x\right)^{3}$ condoning the omission of brackets.

Accept any coefficient appearing from Pascal's triangle. FYI 10, 45, 120

A1 For any two non constant terms of $1-\frac{5}{4}x+\frac{45}{64}x^{2}-\frac{15}{64}x^{3}$

A1 For all four terms $1-\frac{5}{4}x+\frac{45}{64}x^{2}-\frac{15}{64}x^{3}$ ignoring terms with greater powers

(b)

M1 For attempting to find $10b+3c$ for their $(3+10x)\left(1+ax+bx^{2}+cx^{3}+\right)$

A1 For $\frac{405}{64}$

Question Number	Scheme	Marks
2	$y=2x^{\frac{3}{2}}-16x^{-2}-6x+9$	B1
	$\int 2x^{\frac{3}{2}}-16x^{-2}-6x+9\,\mathrm{d}x=\frac{4}{5}x^{\frac{5}{2}}+\frac{16}{x}-3x^{2}+9x$	M1 A1 A1
	$\int_{4}^{9}\left(\frac{4}{5}x^{\frac{5}{2}}+\frac{16}{x}-3x^{2}+9x\right)\mathrm{d}x=\left(\frac{4}{5}\times 9^{\frac{5}{2}}+\frac{16}{9}-3\times 9^{2}+9\times 9\right)-\left(\frac{4}{5}\times 4^{\frac{5}{2}}+\frac{16}{4}-3\times 4^{2}+9\times 4\right)$	dM1
	$=16\,{}^{26}\!/_{45}$	A1
		(6) **(6 marks)**

B1	For $y=2x^{\frac{3}{2}}-16x^{-2}-6x+9$
M1	Any correct index
A1	Two correct terms of (may be un-simplified) $\frac{4}{5}x^{\frac{5}{2}}+\frac{16}{x}-3x^{2}+9x$
A1	All terms correct of (may be un-simplified) $\frac{4}{5}x^{\frac{5}{2}}+\frac{16}{x}-3x^{2}+9x$
dM1	Substitutes 9 and 4 into their integral and subtracts either way around
A1	$16\,{}^{26}\!/_{45}$

Question Number	Scheme	Marks
3.(a)	$a+5d=11$ and $\frac{3}{2}\{2a+2d\}=-15$	B1 B1
	Solves simultaneously $\Rightarrow d=4, a=-9*$	M1 A1
		(4)
(b)	Attempts $\frac{n}{2}\{2\times -9+(n-1)\times 4\}...888$	M1
	$2n^2-11n-888...0 \Rightarrow n=(24)$	M1
	$n=25$	A1
		(3)
		(7 marks)

(a)

B1 One of $a+5d=11$ or $\frac{3}{2}\{2a+3d\}=-15$

For the sum you may see $a+a+d+a+2d=-15$

B1 Both formulae correct. May be un-simplified

M1 Solves their two formulae (one of which must be correct) and finds at least *a* or *d*

A1 For $d=4, a=-9*$

(b)

M1 Attempts to use the AP $\frac{n}{2}\{2\times -9+(n-1)\times 4\}...888$ with their *a* and *d*

M1 Solves their equation for *n*. Allow trial and improvement/ calculator /factoristion /formulae.

FYI the correct equations are $n(2n-11)=888$ $2n^2-11n-888=0$

A1 $n=25$ (The $n=-18.5$ if found must not be given as a possible answer)

Question Number	Scheme	Marks
	$\mathrm{f}(x)=2x^3+\frac{3}{2}x^2-18x+3$	
4.(a)	$\mathrm{f}'(x)=6x^2+3x-18$	B1
	Sets $\mathrm{f}'(x)=0 \Rightarrow 6x^2+3x-18=0 \Rightarrow x=-2,\frac{3}{2}$	M1 A1
	$-2<x<\frac{3}{2}$	A1 ft
		(4)
(b)	Attempts to find $\mathrm{f}(-2)$ or $\mathrm{f}\left(\frac{3}{2}\right)$	M1
	Finds the local max and min values 29 and -13.875	A1
	$k<-13.875,\quad k>29$	A1ft
		(3) **(7 marks)**

(a)

B1 $\mathrm{f}'(x)=6x^2+3x-18$ which may be un-simplified

M1 Sets their $\mathrm{f}'(x)=0$ and attempts to solve using any allowable method including use of a calculator

A1 $x=-2,\frac{3}{2}$

A1 ft $-2<x<\frac{3}{2}$ or $-2 \leqslant x \leqslant \frac{3}{2}$ but follow through on the inside region for their $x=-2,\frac{3}{2}$

(b)

M1 Attempts to find the y value at either of their turning points found in (a) $\mathrm{f}(-2)$ or $\mathrm{f}\left(\frac{3}{2}\right)$

A1 Finds the local maximum and minimum values of 29 and -13.875

A1ft $k<-13.875,\quad k>29$ but follow through on the outside region for their -13.875 and 29

Question Number	Scheme	Marks
5.(a)	Sets $\mathrm{f}(\pm 2) = -24 \rightarrow$ equation in a Eg. $-8+16+a=-24$ $\Rightarrow a=-32$ *	M1 A1* **(2)**
(b) (i)	$x^3-8x-32=(x-4)(x^2+4x+8)$	M1 A1
(ii)	Attempts to find roots of $x^2+4x+8=0 \Rightarrow (x+2)^2=-4$	M1
	States that $x^2+4x+8=0$ has no (real) roots as there are no real solutions to $\sqrt{-4}$ so $\mathrm{f}(x)=0$ has only one (real) root, $x=4$	A1 **(4)** **(6 marks)**

(a)

M1 Sets $\mathrm{f}(\pm 2) = -24 \rightarrow$ equation in a Eg. $-8+16+a=-24$. Condone sign slips

A1* Sets $\mathrm{f}(-2) = -24 \rightarrow$ Completes proof with at least one intermediate "solvable" line such as $-8+16+a=-24$

(b)(i)

M1 Attempt to divide or factorise out $(x-4)$

By factorisation look for $x^3-8x-32=(x-4)(x^2+kx\pm 8)$ $k \neq 0$

By division look for

$$\begin{array}{r} x^2 \ \pm 4x.......... \\ x-4\overline{)x^3+0x^2-8x-32} \\ \underline{x^3-4x^2} \end{array}$$

A1 Correct factors $(x-4)(x^2+4x+8)$

(b)(ii)

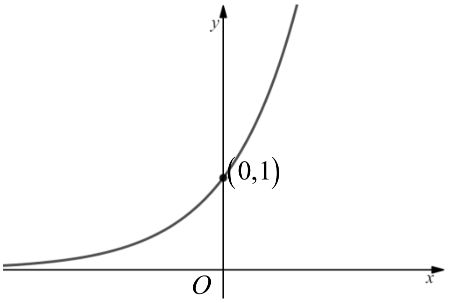
M1 Attempt to find the (number of) roots of their (x^2+4x+8)

Allow completing the square (See scheme)
Allow formula

Allow attempt at b^2-4ac

A1 CSO. This requires

- Correct factors $(x-4)(x^2+4x+8)$
- Correct working leading to conclusion that (x^2+4x+8) has no real roots
- A statement that $\mathrm{f}(x)=0$ has only one real root, $x=4$

Question Number	Scheme	Marks
6.(a)	 Shape or intercept at 1	M1
	Fully correct	A1
		(2)
	$h = 0.4$	B1
(b)	Area $\approx \frac{0.4}{2}\{11+85+2\times(16.37+24.47+36.83+55.8)\}$	M1
	=72.6	A1
		(3)
(c)	$\int_2^4 (3^{x+1}+x)\,\mathrm{d}x = \int_2^4 3\times(3^x+x)-2x\,\mathrm{d}x = 3\times(b)-\int_2^4 2x\,\mathrm{d}x$	M1
	$= 3\times(b)-\left[x^2\right]_2^4$	dM1
	$= 205.8$	A1ft
		(3)
		(8 marks)

(a)
M1 For either the shape (any position) or the y-intercept at 1
A1 Fully correct. Shape in quadrants 1 and 2 only with the y -intercept at 1

(b)

B1 For $h = 0.4$ This is implied by sight of $\frac{0.4}{2}$ in front of the bracket

M1 For a correct bracket condoning slips.
A1 awrt 72.6

(c)

M1 For realising that $\int_2^4 (3^{x+1}+x)\,\mathrm{d}x = \int_2^4 3\times(3^x+x)-2x\,\mathrm{d}x = 3\times(b)\pm\int_2^4 px\,\mathrm{d}x$

dM1 For $3\times(b)\pm\left[\frac{px^2}{2}\right]_2^4$

A1ft awrt 205.8 or follow through on the answer to their $3(b)-12$

Question Number	Scheme		Marks
7.(i)	$\log_2 3+\log_2 2^{x-2}=\log_2 8^x$	$\frac{2^{3x}}{2^{x-2}}=3$	M1
	$\Rightarrow \log_2 3+x-2=3x$	$2^{2x+2}=3\Rightarrow(2x+2)=\log_2 3$	A1
	$\Rightarrow 2x=\log_2 3-2$	$\Rightarrow 2x=\log_2 3-2$	dM1
	$\Rightarrow x=\frac{\log_2 3}{2}-1$	$\Rightarrow x=\frac{\log_2 3}{2}-1$	A1
			(4)
(ii)	$2\log_5(2y+1)-\log_5(2-y)=1$		
	$\log_5\frac{(2y+1)^2}{(2-y)}=1$		M1
	$\frac{(2y+1)^2}{(2-y)}=5$		A1
	$4y^2+9y-9=0\Rightarrow y=\frac{3}{4},-3$		dM1
	States that -3 cannot be a solution as $\log_5(2y+1)$ doesn't exist for $y=-3$		A1
	Only solution is $y=\frac{3}{4}$		**(4)**
			(8 marks)

(i)

M1 Takes logs of both sides and uses one rule correctly.

Either $\log_2 3\times 2^{x-2}=\log_2 3+\log_2 2^{x-2}$ or $\log_2 8^x=x\log_2 8$

Alternatively attempts to collect terms in x together writing 8^x as 2^{3x}

A1 For a correct equation linear equation in x

Allow for $\log_2 3+x-2=3x$

$\log_2 3+x-2=x\log_2 8$

$(2x+2)=\log_2 3$

$(2x+2)\log_2 2=\log_2 3$

dM1 Having achieved a linear equation in x, and used two log rules correctly, it is scored for making x the subject. The solution may be still in terms of $\log_2 8$

A1 $x=\frac{\log_2 3}{2}-1$ or exact equivalent such as $x=\frac{\log_2 3-2}{2}$.

(ii)

M1 Uses two log laws Eg. $2\log_5(2y+1)-\log_5(2-y)=\log_5\frac{(2y+1)^2}{(2-y)}$

A1 A correct equation in y

dM1 Dependent upon having scored the M1. It is for proceeding to find at least one value for y using a correct method including use of a calculator.

A1 States that -3 cannot be a solution as $\log_5(2y+1)$ is not defined for/doesn't exist for $y=-3$

Hence only solution is $y=\frac{3}{4}$

Question Number	Scheme	Marks
8 (a)	$t=1\Rightarrow H=3-1.2\sin\left(\frac{\pi}{6}\right)=2.4\text{ (m)}$ *	B1*
		(1)
(b)	Maximum = 4.2 m	B1
		(1)
(c)	$H=3.5\Rightarrow 3.5=3-1.2\sin\left(\frac{\pi t}{6}\right)$	
	$\Rightarrow \sin\left(\frac{\pi t}{6}\right)=-\frac{5}{12}$	M1 A1
	$\Rightarrow \sin\left(\frac{\pi t}{6}\right)=-\frac{5}{12}\Rightarrow t=\frac{6\times\left(\pi+\arcsin\left('\frac{5}{12}'\right)\right)}{\pi}=6.82(081)\to 11:49\text{ am}$	M1A1
	And $\Rightarrow t=\frac{6\times\left(2\pi-\arcsin\left('\frac{5}{12}'\right)\right)}{\pi}=11.17(919)=4:11\text{ pm}$	M1A1
		(6)
		(8 marks)

(a)

B1* This is a given answer. Score for sight of $3-1.2\sin\left(\frac{\pi}{6}\right)=2.4\text{ (m)}$ OR $3-1.2\times\frac{1}{2}=2.4\text{ (m)}$

(b)

B1 For 4.2 (m)

(c)

M1 For substituting H=3.5 into $H=3-1.2\sin\left(\frac{\pi t}{6}\right)$ WITH some attempt to make $\sin\left(\frac{\pi t}{6}\right)$ the subject.

You may see the $\left(\frac{\pi t}{6}\right)$ being replaced by another variable which is fine for the first two marks

A1 $\sin\left(\frac{\pi t}{6}\right)=-\frac{5}{12}$ oe Condone awrt $\sin\left(\frac{\pi t}{6}\right)=-0.417$

M1 For a correct attempt to find one of the first two values of t using their $-\frac{5}{12}$.

Either $t=\frac{6\times\left(2\pi-\arcsin\left('\frac{5}{12}'\right)\right)}{\pi}$ or $\frac{6\times\left(\pi+\arcsin\left('\frac{5}{12}'\right)\right)}{\pi}$

A1 One correct value of t awrt 6.8 or awrt 11.2

M1 For a (correct) attempt to find the second value of t using their $-\frac{5}{12}$

A1 Both 11:49 am and 4:11 pm

Note: Some candidates may choose to do part (c) using degrees. This is fine and the scheme can be applied

M1 A1 $\sin(30t)=-\frac{5}{12}$

M1 Attempts to find one value of t (but the units must be consistent)

$$\sin(30t)=-\frac{5}{12}\Rightarrow 30t=204.6\text{ or }335.4\Rightarrow t=$$

Question Number	Scheme	Marks
9.(a)	Uses common ratio's $\Rightarrow \dfrac{\cos\theta}{\sin\theta} = \dfrac{0.5}{\cos\theta}$	B1
	Uses $\cos^2\theta = 1-\sin^2\theta$ $\cos^2\theta = 0.5\sin\theta \Rightarrow 1-\sin^2\theta = 0.5\sin\theta$	M1
	$2\sin^2\theta + \sin\theta - 2 = 0$ *	A1*
		(3)
(b)	Attempts to solve $2\sin^2\theta + \sin\theta - 2 = 0 \Rightarrow \sin\theta = \dfrac{-1\pm\sqrt{17}}{4}$	M1
	$\sin\theta = \dfrac{-1+\sqrt{17}}{4}$	A1
	Solves by using arcsin to find solution(s)	M1
	$\theta = 128.7°$	A1
		(4)
(c)	Attempts $r = \dfrac{\cos\theta}{\sin\theta}$ with $\theta = 128.7° \Rightarrow r = -0.80$	M1 A1
	States that as $\lvert r\rvert < 1$ the series is convergent	B1
(d)		**(3)**
	(ii) Attempts $S_\infty = \dfrac{\sin\theta}{1-"r"} = 0.43$	M1 A1
		(2)
		(12 marks)

(a)

B1 Uses common ratios to produce a correct equation usually $\dfrac{\cos\theta}{\sin\theta} = \dfrac{0.5}{\cos\theta}$ or $\cos\theta \times \dfrac{\cos\theta}{\sin\theta} = 0.5$

M1 Uses $\cos^2\theta = 1-\sin^2\theta$ $\cos^2\theta = 0.5\sin\theta \Rightarrow 1-\sin^2\theta = 0.5\sin\theta$

A1* cso $2\sin^2\theta + \sin\theta - 2 = 0$

(b)

M1 Attempts to solve $2\sin^2\theta + \sin\theta - 2 = 0$ using the formula or calculator

A1 Achieves $\sin\theta = \dfrac{-1+\sqrt{17}}{4}$ or awrt 0.78

M1 Uses arcsin to find at least one solution

A1 awrt $\theta = 128.7°$ only

(c)

M1 Attempts to find $r = \dfrac{\cos\theta}{\sin\theta}$ with $\theta = 128.7° \Rightarrow r = ...$

Alternatively uses $r = \dfrac{0.5}{\cos\theta}$ with $\theta = 128.7° \Rightarrow r = ...$

A1 awrt $r = -0.80$

B1 States as $\lvert r\rvert < 1$ the series is convergent

Alternatively as $-1 < -0.80 < 1$ the series is convergent

(d)

M1 Attempts $S_\infty = \dfrac{\sin\theta}{1-"r"} =$

A1 awrt 0.43

Question Number	Scheme	Marks
10(a)	$M=(7,0)$	B1
	Attempts gradient of $PQ=\frac{6-(-6)}{4-10}=(-2)$	M1
	Equation of l is $y-0=\frac{1}{2}(x-7)$	dM1
	$x-2y-7=0$	A1
		(4)
(b)	Substitutes $y=-2$ into $y=\frac{1}{2}x-\frac{7}{2}\Rightarrow x=$	M1
	Obtains centre of circle $(3,-2)$	A1
	Attempts $r^2=(4-"3")^2+(6-"-2")^2$	M1
	$(x-3)^2+(y+2)^2=65$	A1
		(4)
		(8 marks)

(a)

B1 States or implies that $M=(7,0)$

M1 Attempts to find the gradient of PQ Required to find $\frac{\Delta y}{\Delta x}$

M1 Uses $M=(7,0)$ and the negative reciprocal gradient to find equation of line l

A1 $x-2y-7=0$ or any multiple thereof.

(b)

M1 Attempts to find the x coordinate of C by substituting $y=-2$ into l

A1 For $C=(3,-2)$

It may also be awarded for a circle equation in the form $(x-3)^2+(y+2)^2=k,\quad k>0$

M1 Attempts to find the distance between their $(3,-2)$ and either $P(4,6)$ or $Q(10,-6)$

A1 $(x-3)^2+(y+2)^2=65$